Total No. of Printed Pages: 3

SUBJECT CODE NO:- B-2039
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F.Y. (Sem-I)
Examination November/December- 2022
Mathematics MAT – 101
Differential Calculus

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all question
2) Figure to the right indicate full marks.

Q. 1 (A) Attempt any one: 08

(a) If U and V be two functions of x possessing derivatives of the nth order then prove that,

$(UV)_n = Un + nC_1\ U_{n-1}\ V_1 + nC_2\ U_{n-2}\ V_2 + \_\_\_\_\_ + \_\_\_\_\_ + nC_r\ U_{n-r}\ V_r + \_\_\_\_\_ + {}^nC_n\ UV_n$

(b) Show that, if fis finitely derivable at c, then f is also continuous at c.

(B) Attempt any one: 07

(c) If $f(x)=x^2 \sin(1/x)$ when $x \neq o$ and f(o), show that f is derivable for every value of x but the derivative is not continuous for x=o

(d) If y= a cos (log x)+b sin (log x). show that,

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

Q. 2 (A) Attempt any one: 08

(a) If a function f is,

i. Continuous in closed interval [a,b]

ii. Derivable in the open interval (a,b)

iii. f(a)=f(b), Then, Prove that, three exists at least one value $c \in (a, b)$ such that, $f^1(c)=0$

(b) If $z=f(x_1y)$ is homogeneous function of x,y of degree n, then prove that,

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x\, \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

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(B) Attempt any one:

(c) Discuss applicability of Rolle's theorem to the function f(x) $= |x|\ in\ [-1,1]$ 07

(d) If z=(x+y) ∅ $(\frac{y}{x})$, where ∅ is any x $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

Q. 3 (A) Attempt any one: 05

(a) Prove that, the gradient of scalar point function is a vector point function

(b) Prove that, grad f(r) x $\underset{r}{\rightarrow} = 0$ where, $r = \sqrt{x^2 + y^2 + z^2}$ and $\underset{r}{\rightarrow} = x\bar{\imath} + y\bar{\imath} + z\bar{k}$

(B) Attempt any one:

(c) Show that, 05

Grad $(\underset{f}{\rightarrow}.\underset{g}{\rightarrow}) = \underset{f\times}{\rightarrow} curl\ \underset{g}{\rightarrow} + \underset{g}{\rightarrow}\times curl\ \underset{f}{\rightarrow} + \left(\underset{f}{\rightarrow}\nabla\right)\underset{g}{\rightarrow} + (\underset{g}{\rightarrow}.\nabla)\underset{f}{\rightarrow}$

(d) Show that $\forall\ x\ \in R$

Sin x=$x - \frac{x^3}{3!} + \frac{x^5}{5!} +$ _____

Q. 4 Choose the correct alternative. 10

i. $\lim_{x\to o} \sin(\frac{1}{x})$ _____

a) Exists
b) Is equal to zero
c) Is equal to ∞
d) Does not exists

ii. If $x^p\ y^q = (x+y)^{p+q}$ Then, $\frac{dy}{dx}$ is equal to_____

a) $\frac{y}{x}$
b) $\frac{py}{qx}$
c) $\frac{x}{y}$
d) $\frac{qy}{px}$

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iii. If x=t-sin t, y=1-cost, Then $\frac{d^2y}{dx^2}$ at $(\pi, 2)$ will be_____

a) 0

b) 1

c) π

d) ∞

iv. If f is continuous in [a,b] and differentiable in (a,b) then three exists at least one point C in (a,b) such that $f^1(c)$ is equal to_____

a) $\frac{f(b)+f(a)}{b+a}$

b) $\frac{f(b)-f(a)}{b+a}$

c) $\frac{f(b)-f(a)}{b-a}$

d) $\frac{f(b)+f(a)}{b-a}$

v. curl $\underset{r}{\rightarrow}$=_____

a) 1

b) 2

c) 3

d) 0

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SUBJECT CODE NO:- B-2040
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F. Y. (Sem-I)
Examination November/December- 2022
Mathematics MAT - 102
(Differential Equations)

[Time: 1:30 Hours] **[Max. Marks:50]**

"Please check whether you have got the right question paper."

N.B.
1) Attempt all questions.
2) Figures to the right indicates full marks.

Q.1 A) Attempt any one. 08

a) Explain the method of Solving differential equation $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x or constants.

b) Explain the method of solving differential equation

$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = X,$

Where $P_1, P_2, \ldots, P_n$ are constants and X is a function of x.

B) Attempt any one. 07

c) Solve the simultaneous equations

$\frac{dx}{dt} - 7x + y = 0 \; ; \frac{dy}{dt} - 2x - 5y = 0$

d) Solve $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + y = 2e^{2x}$

Q.2 A) Attempt any one. 08

a) Explain the method of solving the differential equation

$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + P_{n-1} x \frac{dy}{dx} + P_n y = X,$

Where $P_1, P_2, P_3 \ldots, P_n$ are constants and X is a function of x.

b) Solve $x^2 \frac{d^2 y}{dx^2} + 7x \cdot \frac{dy}{dx} + 5y = x^5$

B) Attempt any one. 07

c) Solve $\frac{d^2 y}{dx^2} - 4y = 2 \cdot \sin\left(\frac{1}{2} \cdot x\right)$

d) Solve $(5 + 2x)\frac{d^2 y}{dx^2} - 6(5 + 2x)\frac{dy}{dx} + 8y = 0$

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Q.3 A) Attempt any one. 05

a) Explain the method of solving equation $\frac{d^n y}{dx^n} = f(x)$

b) Derive the Partial differential equation by the elimination of the arbitrary constants from the equation $\phi(x, y, z, a, b) = 0$.

B) Attempt any one. 05

c) Solve $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$

d) Form a Partial differential equation by eliminating the arbitrary function from $z = F(x^2 + y^2)$.

Q.4 Choose correct alternative. 10

i) The integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ is _____.

(a) $e^{\int P dx}$

(b) $e^{-\int P dx}$

(c) e^x

(d) e^{Px}

ii) The general solution of the differential equation $\frac{d^2y}{dx^2} - a^2 y = 0$ is _____.

(a) $y = (c_1 + c_2 x)e^{ax}$

(b) $y = (c_1 + c_2 x)e^{-ax}$

(c) $y = c_1 e^{ax} + c_2 e^{-ax}$

(d) None of these

iii) The particular integral of the differential equation $\frac{d^2y}{dx^2} - y = 2 + 5x$ is _____.

(a) $2 + 5x$

(b) $-2 - 5x$

(c) $-2 + 5x$

(d) $2 - 5x$

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iv) The Solution of the Simultaneous equation $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is _____.

(a) $x = c_1 y$ and $x = c_2 z$

(b) $x = c_1 y^2$ and $x = c_2 z^2$

(c) $x = c_1 x^2$ and $x = c_2 z^2$

(d) None of the above

v) The Partial differential equation corresponding to the equation $z = (x + a)(y + b)$ is _____.

(a) $z = p^2 q^2$

(b) $z = p + q$

(c) $z = p - q$

(d) $z = pq$

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SUBJECT CODE NO:- B-2054
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F. Y. (Sem-II)
Examination November/December- 2022
Mathematics MAT - 201
(Integral Calculus)

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B i) Attempt all the Questions.
ii) Figures to the right indicate full marks.

Q.1 A) Attempt any one. 08

a) Obtain a reduction formula for $\int x^m (\log x)^n dx$ and evaluate $\int_0^1 x^4 (\log x)^3 dx$.

b) Obtain reduction formula for $\int \cos^n x \, dx$. Where n is positive integer. Also find $\int_0^{\pi/2} \cos^8 x \, dx$.

B) Attempt any one. 07

c) Evaluate $\int_2^3 \frac{(x^2+1)}{(2x+1)(x^2-1)} dx$.

d) Evaluate $\int \frac{dx}{1-x^6}$

Q.2 A) Attempt any one 08

a) Evaluate $\int_a^b \sin hx \, dx$ as the limit of sum.

b) Find the area between the curve

$$x(x^2 + y^2) = a(x^2 - y^2)$$

and its asymptote. Also find the area of its loop.

B) Attempt any one 07

c) Find the perimeter of the loop of the curve

$$9\, ay^2 = (x - 2a)(x - 5a)^3$$

d) Find the volume of the solid obtained by revolving the cardioide

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$r = (1 + \cos\theta)$ about the initial line.

Q.3 A) Attempt any one 05

a) Show that

$$\frac{1}{3}\int_s \vec{r}\,.\,d\vec{a} = V$$

Where V is the volume enclosed by the surface S.

b) Verify stoke's theorem for the function $\vec{F} = x(\vec{\imath}x + \vec{\jmath}y)$, integrated round the square in the plane $z = 0$ whose sides are along the line.

$x = 0, y = 0, x = a, y = a$

B) Attempt any one 05

c) Evaluate $\int_c \vec{F}\,.\,d\vec{r}$ where

$\vec{F} = (2y + 3)\vec{\imath} + xz\vec{j} + (yz - x)\vec{k}$

Along the path C is straight line joining (0,0,0) to (2,1,1)

d) Evaluate $\int_s \frac{\vec{r}}{r^3} d\vec{a}$

Where S denotes the sphere of radius a with center at the origin.

Q.4 Choose the correct alternatives. 10

1) $\int \frac{dx}{3x-4} =$ ____

a) $3\log(3x-4)$ b) $^1/_3 \log(3x)$

c) $^1/_3 \log(3x-4)$ d) $^1/_4 \log(3x-4)$

2) $\int^{\pi/_2} \sin^8 x\, dx$

a) $\frac{35\pi}{256}$ b) $\frac{256\pi}{35}$ c) $\frac{35}{256\pi}$ d) $\frac{256}{35\pi}$

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3) The perimeter of the curve $r = 2\cos\theta$ is _____.

a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) 2π

4) The volume generated by revolving about the x-axis an area bounded by the curve $y = t(x)$ and the two ordinates $x = a$ and $y = b$ is given by

a) $\int_a^b y^2 dx$ b) $1/2 \int_a^b y^2 dx$

c) $\frac{\pi}{2}\int_a^b y^2 dx$ d) $\pi \int_a^b y^2 dx$

5) Value of $\int(xdy - ydx)$ around the circle $x^2 + y^2 = 1$ is

a) 0 b) $\frac{\pi}{2}$ c) π d) 2π

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Total No. of Printed Pages: 03

SUBJECT CODE NO:- B-2055
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F.Y (Sem-II)
Examination November/December- 2022
Mathematics MAT - 202
(Geometry)

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) Attempt all questions.
ii) figures to the right indicate full marks.

Q. 1 A) Attempt any one: 08

a) Show that the equation of the first degree in x, y, z represents a plane.

b) Find the angle between the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax + by + cz + d = 0$.

B) Attempt any one: 07

c) Find the equation of the plane passing through the lines of intersection of the planes $2x - y = 0$ and $3z - y = 0$ are perpendicular to the plane $4x + 5y - 3z = 8$.

d) Find the equation of the plane containing the line $2x - 5y + 2z = 6, 2x + 3y - z = 5$ and parallel to the line $x = \frac{-y}{6} = \frac{z}{7}$.

Q. 2 A) Attempt any one: 08

a) Find the condition that two straight lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}, \frac{x-z_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are coplanar.

b) Define a sphere, obtain the equation to a sphere on line joining the point (x_1, y_1, z_1) and (x_2, y_2, z_2) as a diameter.

B) Attempt any one: 07

a) Find the equation of the line through the point (1,2,3) parallel to the line $x - y + 2z = 5, 3x + y + z = 6$.

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b) Show that the two spheres

$x^2 + y^2 + z^2 - y + 2z = 0\,, x - y + z - 2 = 0\,;$

$x^2 + y^2 + z^2 + x - 3y + z - 5 = 0, 2x - y + 4z - 1 = 0$

line on the same sphere and find its equation.

Q. 3 A) Attempt any one 05

a) Prove that every section of a right circular cone by a plane perpendicular to its axis is a circle.

b) Fine the condition that the plane lx+my+nz=p, should touch the central conicoid $ax^2 + by^2 + cz^2 = 1.$

B) Attempt any one 05

c) Find the length of the perpendicular from the point P (5,4, -1) upon the line $\frac{1}{2}(x-1) = \frac{1}{9}y = \frac{1}{5}z.$

d) Fine the equation to the right circular cone whose vertex is at origin, the axis along x-axis and semi-vertical angle is α .

Q.4 Choose the correct alternative: 10

1) The angle between the two planes $3z - 4y + 5z = 0\ and\ 2x - y - 2z = 5\ is$_____

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

2) The foot of perpendicular form (2,3,4) to the plane $x + y - z + 4 = 0$ is _____

a) $\left(\frac{-1}{3}, \frac{4}{3}, \frac{17}{3}\right)$ b) $(1/3, -4/3, \frac{17}{3})$ c) $\left(\frac{1}{3}, \frac{4}{3}, \frac{17}{3}\right)$ d) $(1/3,\ 4/3, -17/3)$

3) Centre of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z + 8 = 0$ is _____

a) (2,-3,4) b) (2,3,4) c) (-2,-3,-4) d) (1,2,3)

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4) The plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ at the point _____.

a) (1,-4,-2) b) (-1,4,-2) c) (-1,4,2) d) (1,4,-2).

5) The straight line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$, if __

a) $al + bm + cn = 1$ c) $al + bm + cn = 0$

b) $ax_1 + by_1 + cz_1 = 1$ d) $ax_1 + by_1 + cz_1 = 0$

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Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2050
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-III)
Examination November/December- 2022
Mathematics MAT – 301
Number Theory

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 a) Attempt any one of the following: 08

i. If k> 0, then prove that gcd (ka, kb) = k gcd(a, b).

ii. For integers a, b, c, prove the following

α) if $a|b$ and $b|c$ then $a|c$,

β) if $a|b$ and $a|c$ then $a|(bx + cy)$ for arbitrary integers x and y

b) Attempt any one of the following: 07

i. If a is odd integer, then prove that $32|(a^2 + 3)(a^2 + 7)$.

ii. Find all solutions in the integers of the Diophantine equation $24x + 138y = 18$.

Q.2 a) Attempt any one of the following: 08

i. State and prove Chinese remainder theorem.

ii. If $p$is prime number, then prove that (p-1)!= -1(modp).

b) Attempt any one of the following: 07

i. Solve the linear congruence 25x = 15(mod29).

ii. If gcd (a, 133) = gcd(b, 133) = 1, then show that $133 \mid a^{18} - b^{18}$.

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Q.3 a) Attempt any one of the following: 05

i. If p is a prime number and $p|ab$, then prove that $p|a$ or $p|b$.

ii. If F is multiplicative function and is defined by

$$F(n) = \sum_{d|n} f(d),$$

then prove that f is multiplicative function.

b) Attempt any one of the following: 05

i. Calculate $\phi(360)$.

ii. Find the values of $\tau(180)$ and $\sigma(180)$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) $\gcd(-12,30) = -------$

a) 6 b) 4 c) 3 d) 1

2) The number of solutions of linear congruence $6x \equiv 15 (\text{mod} 21)$ is ..

a) 6 b) 3 c) 1 d) 15

3) The value of $\mu(10)$is ----------

a) -1 b) 0 c)5 d)1

4) If $\gcd(a,b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right)$ = --------

a) 1 b) d c)$\frac{1}{d}$ d) ab

5) If $a|bc$ with $\gcd(a,b) = 1$ then ----------

a) $b|a$ b) $a|c$ c) $c|a$ d) $a = b$

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Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2051
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-III)
Examination November/December- 2022
Mathematics MAT - 302
Integral Transforms

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B All questions are compulsory ,between internal choice in available
Figures to the right indicate full marks

Q.1 (a) Attempt any one of the following: 08

i. If $L^{-1}\{f(s)\} = F(t)$, then prove that $(L^{-1})f^n(s)\} = (-1)^n t^n F(t)$

ii. Derive the relation between Fourier transform and Laplace transform.

(b) Attempt any one of the following: 07

i. Using Laplace transform, find the solution of the differential equation
$(D^2 + D)y = t^2 + 2t$,
where y(0)=4 $y'(0) = -2$

iii. Find the value of $L^{-1}\left\{\frac{1}{s(s+1)^3}\right\}$

Q.2 a) Attempt any one of the following 08

i. If $L\{F(t)\}=f(s)$,then prove that $\lim_{s\to\infty} F(t) = \lim_{t\to 0} sf(s)$

ii. If $\tilde{f}(s)$and $\tilde{g}(s)$ are Fourier transforms of f(x)and g(x) respectively, then prove that
$F\{a\,f(x) + bg(x)\} = a\,\tilde{f}(s) + b\,\tilde{g}(s)$
Where a and b are constants

b) Attempt any one of the following 07

i. Prove that $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\} = \frac{2}{t}\,sintsinh\,t.$

ii. Using Laplace transform, prove that $\int_0^\infty te^{-3t}sintdt = \frac{3}{50}$

Q.3 (a) Attempt any one of the 05

i. If $L\{F(t)\} = f(s)$, then prove that $L\{e^{at}F(t)\} = f(s + a)$.

ii. If f(s) is the Fourier transform F(x), then prove that the Fourier transform of F'(x) is equal to is f(s).

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(b) Attempt any one of the following: 05

i. Evaluate the integral

$$\int_0^{\infty} e^{ax} x^{m-1} sinbx dx$$

ii. Evaluate L{sin at - at cos at}.

Q.4 Choose the correct alternative and rewrite the sentence: 10

(a) If $\int_0^{\infty} e^{-} dx = \frac{\sqrt{\pi}}{2}, then \int_{-\infty}^{\infty} e^{-x^2 dx} =$_____

i. $\frac{\sqrt{\pi}}{2}$

ii. $\sqrt{\frac{\pi}{2}}$

iii. $\sqrt{\pi}$

iv. 0

(b) $L\{2t^3 - 6t + 8\} =$_____

i. $\frac{12}{s^3} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

ii. $\frac{6}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

iii. $\frac{12}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

iv. $\frac{12}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

(c) $L^{-1}\left\{\frac{1}{s-a}\right\} =$____,$s > a$

i. ae^t

ii. a^{at}

iii. a^{-at}

iv. ae^{-t}

(d) The sine transform of f(x)=$\frac{1}{x}$ is_____

i. $\sqrt{\pi}$

ii. π

iii. 2π

iv. $\frac{\pi}{2}$

(e) $L\{\sinh at\} =$_____

i. $\frac{a}{s^2 - a^2}$

ii. $\frac{s}{s^2 - a^2}$

iii. $\frac{a}{s^2 + a^2}$

iv. $\frac{s}{s^2 + a^2}$

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Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2065
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-IV)
Examination November/December- 2022
Mathematics MAT - 401
Numerical Methods

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) Attempt all questions.
ii) Figure to the right indicate full marks.
iii) Use of non-programmable calculator and logarithmic table is allowed.

Q.1 A) Attempt any one: 08

a) Derive newton – Raphson formula for finding real roots of an equation f(x) = 0.

b) Derive Newton's general interpolation formula.

B) Attempt any one:

c) Obtain a root, correct to four decimal places, which lies between 2 and 3 of the equation $f(x) = x^3-2x -5 = 0$, by Using the method of false position. 07

d) Certain corresponding values of x and $\log_{10}^{x}$ are (300, 2.4771), (304, 2.4829), (305; 2.4843) and (307, 2.4871) Find $\log_{10} 301$.

Q.2 A) Attempt any one: 08

a) Define chebyshev polynomial and prove the recurrence relation
$T_{n+1}(x) = 2x\, T_n(x) – T_{n-1}(x)$.
Where $T_n(x)$ is a chebyshev polynomial of degree n.

b) Explain the Gaussian elimination method for solving system of linear equation.

B) Attempt any one: 07

c) Fit a straight line of the form $Y=a_0 + a_1x$ to the data.

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5.0	6.0

d) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

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Q.3 A) Attempt any one: 05

a) Explain Picard's method of successive approximations to solve the differential equation $y' = f(x, y)$ With the initial condition $y(x_0) - y_0$

b) Prove that the Newton-Raphson method has quadratic convergence.

B) Attempt any one: 05

c) Using Euler's method, solve the differential equation $\frac{dy}{dx} + 2y = 0, \quad y(0) = 1$ take h=0.1 and obtain y(0.1), y(0.2) and y(0.3).

d) Using the method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots + (-1)^n u_{x-n}$$

Q.4 Choose the correct alternative. 10

i) Rate of convergence of Newton-Raphson method is _____
a) Linear b) Quadratic c) Cubic d) Biquadratic

ii) $\Delta^2 y_1 = ---$
a) $y_2-2y_1+y_0$ b) $y_3+2y_2+y_1$ c) $y_3-y_2+y_1$ d) $y_3-2y_2+y_1$

iii) The chebyshev polynomial of degree one is _____
a) x b) $2x^2-1$ c) $2x^2+1$ d) 1

iv) The eigenvalues of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are _____
a) 3,2 b) -3,-2 c) 1,-1 d) 0,4

v) Newton's forward difference interpolation formula is applicable only when the arguments are _____
a) Equally spaced b) Unequally spaced
c)Both equally and unequally spaced d) None of these

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SUBJECT CODE NO:- B-2066
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y (Sem-IV)
Examination November/December- 2022
Mathematics MAT - 402
Partial Differential Equation

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B 1) All questions are compulsory
2) Figures to the right indicate full marks

Q.1 A) Attempt any one 08

i) Explain the method of obtaining complementary function of $(A_o D^n + A_1 D^{n-1}D' + -- + A_n D'^n)z = f(x,y)$

ii) Explain the method of obtaining complete general integral of $f_1(x,p) = f_2(y,q)$

B) Attempt any one: 07

iii) Solve: $x^2p + y^2q = z^2$

iv) Solve :$pz = 1 + q^2$

Q.2 A) Attempt any one: 08

a) Explain Jacobi's method to solve $f(x_1, x_2, x_3, p_1, p_2, p_3) = 0$

b) Discuss Monge's method to solve $Rr + Ss + Tt = V$ where R,S,T and V are functions of x,y,z,p and q

B) Attempt any one 07

c) Solve $(p^2 + q^2)y = qz\ by$ using charpit's method

d) Solve: r+5s +6t=0

Q.3 A) Attempt any one 05

a) With usual notations prove that
$\frac{1}{F(D^2,DD',D'^2)}\cos(ax+by) = \frac{\cos(ax+by)}{F(-a^2,-ab,-b^2)}$; if $F(-a^2,-ab,-b^2) \pm 0$

b) Find the general solution of $(D - mD' - k)z = 0$

B) Attempt any one 05

c) Solve : $\frac{\partial^2 Z}{\partial x \partial y} = \frac{1}{xy}$

d) Solve : $(D^2 - 2DD' + D'^2)Z = e^{x+2y}$

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Q.4 Choose the correct alternatives 10

1) The Lagrange's auxiliary equation of $P_1 \frac{\partial z}{\partial x_1} + P_2 \frac{\partial z}{\partial x_2} + - - - + P_n \frac{\partial z}{\partial x_n} = R$ are ------
   a) $\frac{dx_1}{1} = \frac{dx_2}{1} = \ - - - - = \frac{dx_n}{1}$
   b) $\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = \ - - - - - = \ \frac{dx_n}{P_n}$
   c) $P_1\, dx_1 = P_2 dx_2 = \ - - - - - = P_n dx_n$
   d) None of these

2) The complete integral of $z = px + qy + pq$ is ------
   a) $z = ax + by$ b) $z = ax + ab$ c) $z = ax + by + ab$ d) $z = a + b$

3) The complementary function of $\left(D^2 - 2DD' + D'^2\right)$ z=sin (2x+3y) is ---------
   a) $z = \phi_1\,(y + x) + x\phi_2(y + x)$
   b) $z = \phi_1(y+x) + \phi_2(y + x)$
   c) $z = \phi_1(y - x) + \phi_2(y - x)$
   d) $z = \phi_1(y - x) + x^2\phi_2(y - x)$

4) The value of $\frac{1}{F(D,D')}\, e^{ax+by}$ = -----
   a) $\frac{1}{F(a,b)}\, e^{ax}, if\ F(a,b) \neq 0$
   b) $\frac{1}{F(a,b)}\, e^{by}, if\ F(a,b) = 0$
   c) $\frac{1}{F(a,b)}\, e^{ax+by}, if\ F(a,b) = 0$
   d) $\frac{1}{F(a,b)}\, e^{ax+by}, if\ F(a,b) \neq 0$

5) The direction ratios of the normal at a point (x, y, z) to the surface given by $Pp + Qq = R$ are ---
   a) p,q,1 b) p,q,-1 c) 1,1,1 d) P,Q,R

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Total No. of Printed Pages: 3

SUBJECT CODE NO:- B-2046
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T. Y. (Sem-V)
Examination November/December- 2022
Mathematics MAT - 501
Real Analysis – I

[Time: 1:30 Hours] **[Max. Marks:50]**

"Please check whether you have got the right question paper."

N.B.
1) All questions are compulsory.
2) Figures to the right indicate Full marks.

Q.1 A] Attempt any one:

a) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent. 08

b) Define Cauchy sequence. 08
If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

B] Attempt any one:

c) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers diverging to infinity, then prove that 07
$\lim_{n\to\infty} \text{Sup } S_n = \infty = \lim_{n\to\infty} \inf S_n$.

d) For $n \in I$, let $S_n = \frac{1\cdot 3\cdot 5\cdot \ldots (2n-1)}{2\cdot 4\cdot 6\cdot \ldots 2n}$ 07
Prove that $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n \leq \frac{1}{2}$.

Q.2 A] Attempt any one:

a) Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero real numbers and let $a = \lim_{n\to\infty} \inf \left|\frac{a_{n+1}}{a_n}\right|$, 08
$A = \lim_{n\to\infty} \text{Sup} \left|\frac{a_{n+1}}{a_n}\right|$ Then prove that $\sum_{n=1}^{\infty} |a_n| < \infty$ if $A < 1$.

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b) If $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive numbers then prove that there is a sequence $\{\epsilon_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but For which $\sum_{n=1}^{\infty} \epsilon_n a_n$ still diverges. 08

B] Attempt any one:

a) Does the series 07

i) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ and

ii) $\sum_{n=1}^{\infty} \frac{n+1}{10^{10}(n+2)}$ converge or diverge?

Justify your answer.

b) Prove that the series $\sum_{n=1}^{\infty}(-1)^{n+1} \cdot \frac{1}{n}$ converges. 07

Q.3 A] Attempt any one:

a) If $u_1, u_2, \ldots, u_n$ are implicit functions of $x_1, x_2, \ldots, x_n$ then prove that 05

$$\frac{\partial(u_1, u_2, \ldots, u_n)}{\partial(x_1, x_2, \ldots, x_n)} = (-1)^n \frac{\frac{\partial(F_1, F_2, \ldots, F_n)}{\partial(x_1, x_2, \ldots, x_n)}}{\frac{\partial(F_1, F_2, \ldots, F_n)}{\partial(u_1, u_2, \ldots, u_n)}}$$

b) Prove that the inverse image of the intersection of two sets is the intersection of the inverse images. 05

B] Attempt any one :

c) Find the Jacobian of $y_1, y_2, \ldots, y_n$ being given $y_1 = 1 - x_1$, $y_2 = x_1(1 - x_2)$, $\ldots$, $y_n = x_1 x_2 \ldots x_{n-1}(1 - x_n)$. 05

d) If $x = c \cos u \cos hv$, $y = c \sin u \sin hv$, 05

Prove that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{2} c^2 (\cos 2u - \cos h2u)$.

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Q.4 Choose correct alternative of the following. 10

1) If $f : A \to B$ is a function defined by $f(x) = \sqrt{x}$ then
   a) $A = B = I\mathbb{R}$
   b) $A = I\mathbb{R}$, $B = I\mathbb{R}^4$
   c) $A = I\mathbb{R}^+$, $B = I\mathbb{R}$
   d) $A = I\mathbb{R}^+$, $B = I\mathbb{R}^+$
2) Total number of sequences can be defined whose range set containing either 1 or -1 are _____.
   a) Countable infinite
   b) Uncountable infinite
   c) Two
   d) One
3) If for every $E > 0$, there exist a positive integer N does not depend on ϵ such that $|S_n - L| < \epsilon$ For all $n \geq N$ then _____
   a) All but finite number of terms of $\{S_n\}$ are equal to L
   b) No term of $\{S_n\}$ is equal to L
   c) Sequence diverges to ∞
   d) Sequence diverges to $-\infty$
4) The Series $\sum \frac{1}{n}$ is
   a) Convergent
   b) Divergent
   c) Oscillatory
   d) None of these
5) If $u(x, y) = xy$ and $v(x, y) = x + y$ then Jacobian of u and v is
   a) x
   b) y
   c) $x - y$
   d) $y - x$

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Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2047
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-V)
Examination November/December- 2022
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A. Attempt any one of the following: 08

a. If ϕ is a homomorphism of G onto $\bar{G}$ with kernel K then prove that ${}^{G}/_{K} \approx \bar{G}$.

b. If G is a finite group and H is a subgroup of G then prove that order of H is a divisor of order of G.

B. Attempt any one of the following: 07

a. If H is a subgroup of a group G then show that $\{x \in G \mid x\,h = hx, for\ all\ h \in F\}$ is a subgroup of G.

b. Prove that the subgroup N of a group G is normal subgroup of G if and only if every left coset of N is G is a right coset of N in G.

Q.2 A. Attempt any one of the following: 08

a. If ϕ is a ring homomorphism of R into R then prove that
i) $\phi\ (0) = 0$
ii) $\phi\ (-a) = -\phi(a)$, for every $a \in R$.

b. If R is a commutative ring with unit element whose only ideals are $\{0\}$ and R itself then prove that R is a field.

B. Attempt any one of the following: 07

c. If U is an ideal of a ring R then prove that
[R:U]=$\{x \epsilon R | r.x \epsilon U$ for every $r \epsilon R\}$ is an ideal of R.

d. With usual notations prove that F[x] is an integral domain.

Q.3 A. Attempt any one of the following: 05

a. Show that every subgroup of an abelian group is a normal subgroup.

b. If U is an ideal of R and 1 ϵ U then show that U=R

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B. Attempt any one of the following: 05

c. Show that $x^3 - 9$ is reducible over the field of integers modulo 11.

d. If G is a group then for all a, b ϵ G prove that $(b.a)^{-1} = a^{-1}.b^{-1}$

Q.4 Choose the correct alternative and rewrite the sentence: 10

1. If o(H) divides o(G) and o(H) ≠ o(G) then _____
   a. H is a subgroup of G
   b. G is a subgroup of H
   c. G=H
   d. H may or may not be subgroup of G.

2. If G is the set of all n x n, nonsingular matrices with rational number entries then under matrix multiplication G is
   a. Finite abelian group
   b. Infinite abelian group
   c. Infinite non abelian group
   d. Finite non abelian group

3. The set of all real numbers is not a group under usual multiplication because
   a. The identity does not exist
   b. Multiplication of reals is not associative
   c. Zero has no inverse
   d. Multiplication of reals not satisfy closure property

4. If K is a subgroup of H, H is a subgroup of G and o(K)=2, o(H)=10, o(G)=20 then index of K in G is _____
   a. 2
   b. 10
   c. 20
   d. 40

5. If R is a ring then $(a-b)^2 = ------$
   a. $a^2 - 2ab + b^2$
   b. $a^2 + 2ab + b^2$
   c. $a^2 - ab - ba + b^2$
   d. $a^2 - ab + ba + b^2$

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Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2061
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics MAT-601
Real Analysis-II

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 A. Attempt any one: 08

a) Let $\langle M_1 , P_1 \rangle$ and $\langle M_2 , P_2 \rangle$ be metric space and let $f : M_1 \to M_2$. Then prove that f is continuous on M_1 if and only $f^{-1}(G)$ is open in M_1 whenever G is open in M_2.

b) If E is any subset of a metric space M , then prove that $\bar{E}$ is closed.

B. Attempt any one: 07

c) Show that if ρ *and* σ are both metrics for a set M , then $\rho + \sigma$ is also a metric for M.

d) $If\ f : R^2 \to R^2\ is\ defined\ by\ f(\langle x , y \rangle) = (\langle y , x \rangle) \quad (\langle x , y \rangle) \epsilon\ R^2$,
$show\ that\ f\ is\ continuous\ on\ R^2$.

Q.2 A. Attempt any one: 08

a) Prove that the metric space $\langle M , P \rangle$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

b) Let f(x) be Riemann integrable in every interval and is periodic with 2π as its period , then prove that $\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} f(a+x)dx$ where a is any number.

B. Attempt any one: 07

c) Prove that R^2 is complete.

d) For each
n ϵ I let b_n be the subdivision $\{0 , 1/n , 2/n , \ldots , n/n\}$ of [0 , 1]. $Compute$ $\lim_{n\to\infty} L[f ; \sigma n]$ for the function $f(x) = x^2 (0 \leq x \leq 1)$.

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Q.3 A. Attempt any one: 05

a) Let f be a continuous function from the compact metric space M_1 into the metric space M_2. Then prove that the range $f(M_1)$ of f is also compact.

b) If f is a continuous function on the closed bounded interval [a , b] , and if $\Phi'(x) = f(x)$ $(a \leq x \leq b)$ then prove that $\int_a^b f(x)dx = \Phi(b) - \Phi(a)$.

B. Attempt any one: 05

c) Find the Fourier series of f(x) = x in $[-\pi, \pi]$.

d) $If\ 0 \leq x \leq 1\ show\ that\ \frac{x^2}{\sqrt{2}} \leq \frac{x^2}{\sqrt{1+x}} \leq x^2$.

Q.4 Choose the correct alternative: 10

I) The convergent sequence in a metric space has -----.
a) Unique limit c) Limit ∞
b) Distinct limit d) None of these

II) If $\langle M , P\rangle = R^1\ and\ \langle A , P\rangle = [0 , 1]$, then the open ball $B\left[0 ; \frac{1}{2}\right] in\ R^1$ is the interval -----.
a) $[-\frac{1}{2} , \frac{1}{2}]$ c) $(-\frac{1}{2} , \frac{1}{2}]$
b) $(0 , \frac{1}{2})$ d) $(-\frac{1}{2} , \frac{1}{2})$

III) The metric space [a, b] with absolute-value metric is -------.
a) Only totally bounded c) Bounded
b) Only complete d) Totally bounded and complete

IV) If f is a bounded function on the closed bounded interval [a , b] and σ is any subdivision of [a , b] , then $\int_{-a}^b f(x)dx = ----$.
a) $l . u . b .\cup [f , \sigma]$ c) $l . u . b . L [f ; \sigma]$
b) $g . l . b .\cup [f ; \sigma]$ d) $g . l . b . L[f , \sigma]$

V) For all n = 0 , 1 , 2 , …….., $\int_{-\pi}^{\pi} \cos^2 nx\ dx = -----$.
a) 0 b) π c) $\pi/4$ d) π^2

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Total No. of Printed Pages:3

SUBJECT CODE NO:-B-2062
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics MAT - 602
Abstract Algebra - II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) All questions are compulsory
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

(a) Prove that if U is a vector space over F and W is a subspace of U, then there is a homomorphism of U onto U/W.

(b) If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional and dim W ≤ dim V.

(B) Attempt any one: 07

(c) Prove that the intersection of two subspaces of a vector space V is a subspace of V.

(d) If W_1 and W_2 are subspaces of finite-dimensional vector space V over F, then show that

$$A(W_1 + W_2) = A(W_1) \cap A(W_2).$$

Q.2 (A) Attempt any one: 08

(a) Prove that a homomorphism T of an R-module M into an R-module N with kernel K(T) is an isomorphism if and only if K(T)=(O).

(b) If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that V=W+$W^{\perp}$.

(B) Attempt any one: 07

(c) If S is subset of a vector space V, let $A(S) = \{f \in \hat{V} | f(s) = 0 \text{ for all } s \in S\}$. Prove that A(S)=A(L(S)), where L(S) is the linear span of S.

(d) If F is the real field and V is $F^{(3)}$, show that the Schwarz inequality implies that the cosine of an angle is of absolute value at most one.

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Q.3 (A) Attempt any one: 05

(a) If S is nonempty subset of the vector space V, then prove that L(S) is a subspace of V.

(b) If u, v ∈ V and $\alpha, \beta \in F$, then prove that $\|\alpha u + \beta v\|^2 = |\alpha|^2\|u\|^2 + \alpha\bar{\beta}(u,v) + \bar{\alpha}\beta(v,u) + |\beta|^2\|v\|^2$.

Where V is an inner product space over F.

05

(B) Attempt any one:

(c) Show that in $F^{(3)}$ the vectors

(1,0,0), (0,1,0) (0,0,1) are linearly independent.

(d) If V is finite-dimensional and $V_1 \neq V_2$ are in V, prove that there is an $f \in \hat{V}$ such that $f(V_1) \neq f(V_2)$.

Q.4 Choose the correct alternative: 10

(i) If V is vector space over a field F, then the subspace V itself and (O) of V are called _____.

(a) Proper subspaces (b) Improper subspaces

(c) Modules (d) None of these

(ii) If W is subspace of a vector space V over F such that dim V=8 and dim W=5, then dim A(W) _____.

(a) 13 (b) 8

(c) 3 (d) 5

(iii) The number of elements in two basis of a finite dimensional vector space is ------

(a) Equal (b) Unequal

(c) May or may not be equal (d) None of these

(iv) The dimension of a vector space R^3 over R is _____.

(a) 2 (b) 4

(c) 1 (d) 3

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(v) An orthogonal set of non-zero vectors is _____.

(a) Linearly dependent

(b) Linearly independent

(c) A basis

(d) None of these

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Total No. of Printed Pages: 3

SUBJECT CODE NO: - CB-2339
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I)
Examination December/January-2022-23
Mathematics - I MAT-011
Geometry

[Time: 1:30 Hours] **[Max. Marks: 40]**

Please check whether you have got the right question paper.

N. B 1) Attempt all questions.
2) Figures to the sight indicate full marks.

Q1 A) Attempt any one: **05**

a) Prove that every equation of the first degree in x, y, z represents a plane.

b) Find the perpendicular distance of the point $p(x_1, y_1, z_1)$ from the plane
$$lx + my + nz = p$$

B) Attempt any one: **05**

c) Find the equation of the plane passing through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4x + 5 = 0$ and the point (1, 1, 1).

d) Show that the distance between the Parallel planes
$2x - 2y + x + 3 = 0$ and $4x - 4y + 2x + 5 = 0$ is $\frac{1}{6}$

Q2 A) Attempt any one: **05**

a) Find the angle between the line
$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$
and the plane
$$ax + by + cz + d = 0.$$

b) Find the magnitude and the equations of the line of shortest distance between the straight lines?
$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$
$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

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B) Attempt any one: **05**

c) Find the co-ordinates of the point of intersection of the line with the plane

$$\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{-2}$$

With the plane

$$3x + 4y + 52 = 5.$$

d) Find the equation of the plane containing the line

$$\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-4}{-2}$$

and the point (0, 6, 0).

Q3 A) Attempt any one: **05**

a) Prove that a plane section of a sphere is a circle.

b) Find the equation of the right circular cone whose vertex is the point (α, β, γ) and whose axis is the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

and semi-vertical angle θ.

B) Attempt any one: **05**

c) Find the radius and centre of the sphere

$$x^2 + y^2 + z^2 - 2x = 4y - 6z = 2$$

d) Find the equation of the radius 2 whose axis passes through the point (1, 2, 3) and has direction cosines proportional to (2,-3, 6).

Q4 Choose the correct alternatives: **10**

1) The intercepts of the plane 2x-3y+4z=12 on the co-ordinates axes are ----------
   a) 2, -3, 4 b) 6, -4, -3 c) 3, -2, 1 d) 6, -4, 3

2) If the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is parallel to the plane $ax = by + cz + d = 0$, then ----------
   a) $al + bm + cn = 0$ b) $am + bn + cl = 0$
   c) $an = bl + cm = 0$ d) $l^2 + m^2 + n^2 = 0$

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3) The direction cosines of the straight line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z-5}{-2}$ are ----------

a) 2, 1, -2 b) $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$ c) $\frac{2}{6}, \frac{1}{6}, \frac{-2}{6}$ d) 2, 2, -1

4) The two equations $x^2 + y^2 + z^2 + 24x + 2vy + 2wz + d = 0$ and $lx + my + nz = p$ taken together represents a ----------

a) Sphere b) Plane c) Pair of lines d) Circle

5) If the vertex of the cone is at origin and the axis is z-axis then equation of cone with semi-vertical angle θ is ----------

a) $x^2 + y^2 = z^2 tan^2\theta$ b) $y^2 + z^2 = x^2 tan^2\theta$

b) $(y^2 + z^2)\tan^2\theta = x^2$ c) $(x^2 + y^2)\tan^2\theta = z^2$

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Total No. of Printed Pages: 2

SUBJECT CODE NO: - CB-2340
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I)
Examination December/January-2022-23
Mathematics -II MAT - 021 Deferential Calculus

[Time: 1.30 Hours] **[Max. Marks:40]**

Please check whether you have got the right question paper.

N. B 1) Attempt all questions
2) Figures to the right indicate full marks.

Q1 A) Attempt any one 05

a) Prove that the union of an arbitrary family of open sets is open.

b) Prove that every convergent sequence is bounded.

B) Attempt any one: 05

a) Show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$

b) Examine the continuity of the function

$$f(x) = \begin{cases} \dfrac{\sin(x-c)}{(x-c)} & ,if\ x \neq c \\ 0 & ,if\ x = c \end{cases}$$

$at\ \ x = c$

Q2 A) Attempt any one 05

a) If $y = \sin(ax+b)$, then prove that $\frac{d^n y}{dx^n} = a^n \sin\left(ax + b + n\frac{\pi}{2}\right)$

b) If a function f defined on [a,b] is continuous on [a,b] and derivable on] a, b [, then prove that there exist at least one real number C between a and b such that

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

B) Attempt any one

05

c) If $y = \log(sinx)$, then prove that $y_3 = \frac{2\cos x}{\sin^3 x}$

d) Discuss the derivability of the function $f(x) = |x| + |x-1|\ at$ x=0

Q3 A) attempt any one 05

a) If ϕ and ψ are two scalar point functions then prove that grad $(\phi + \psi)$= grad ϕ + grad ψ

b) If ϕ is a scalar point function and $\vec{u}$ is a vector point function then prove that

$$div\ (\phi\ \vec{u}) = (\ grad\ \phi)\vec{u} + \phi(\text{div}\ \vec{u})$$

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B) Attempt any one

c) If $\phi(x,y,z) = 3x^2y - y^3z^2$ find grad ϕ at the point $(1,-2,-1)$ 05

d) Find $\frac{d^n y}{dx^n}, if\ y = x^3\ cosx$

Q4 Choose the correct alternative 10

1) If $S_n = (-1)^n, n \in N$,then upper limit of S_n= ----------

a) -1 b) 1 c) 0 d) ∞

2) Limit point of the set $\left\{\frac{1}{n}, n \in N\right\}$ is -------

a) 0 b) -1 c) 1 d) $\frac{1}{2}$

3) The vector differential operator del $(\nabla) = \ - - - - -$

a) $\vec{\imath}\, dx + \vec{j} dy + \vec{k}\, dz$ b) $\partial x + \partial y + \partial z$

c) $\vec{\imath}\frac{\partial}{\partial x} + \vec{j}\ \frac{\partial}{\partial y} + \vec{k}\ \frac{\partial}{\partial z}$ d) $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

4) If $y = \frac{logx}{x}, then\ \frac{d^2y}{dx^2}$= ----------

a) $\frac{2logx}{x}$ b) $\frac{logx}{2x}$ c) $2logx + 3$ d) $\frac{2logx-3}{x^3}$

5) $\lim_{x\to 0-} \frac{|x|}{x} = - - - - -$

a) -1 b) 0 c) 1 d) does not exists

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