



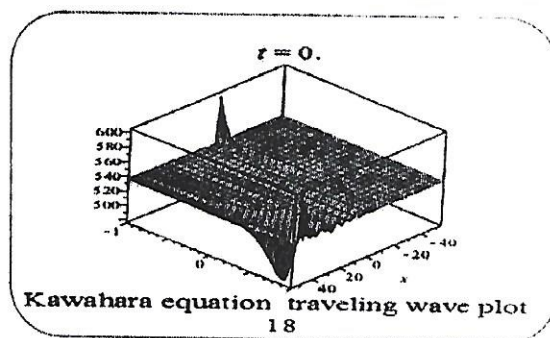
## VARIOUS EXPANSION METHODS FOR FEW FRACTIONAL DIFFERENTIALS AS SOLUTION

Madhuri N. Gadsing  
Department of Mathematics,  
Jawahar Arts, Science and Commerce College, Anadur, District Osmanabad.

### ABSTRACT :

In this paper, we formulate exact solutions of some nonlinear space time fractional differential equations generated by mathematical physics and modified Reimann Liouville in applied mathematics; that is, the fractional modified Benjamin Bona Mahony (MBBM) and the Kawahara equation.

KEYWORDS : Fractional calculus, Kawahara equation.



### INTRODUCTION :

Fractional calculus has been used for the physical and engineering science models. Fractional differential equations (FDEs) are considered as models of physical systems. We introduce the space-time fractional MBBM equation (Equation 1, 2, 3).

$$D_t^\alpha u + D_t^\alpha u - \nu u^2 D_x^\alpha u + D_x^{3\alpha} u = 0 \tag{1}$$

Where  $\nu$  is a nonzero positive constant, we also consider the time fraction mode nonlinear shear defined in equation 4

$$D_t^\alpha u + u^2 u_x + p u_{xx} + q u_{xxx} = 0 \tag{2}$$

Where  $\alpha$  is the parameter for the order of the fractional time derivative, and  $0 < \alpha \leq 1$ : the modified Riemann-Liouville order  $\alpha$  derivative of the Jumaries defined in equation 5

The  $\frac{c}{c}$  expansion method for FDE's

We are considering the following types of common nonlinear fractional differential equations (FDEs)...

$$(u, D_t^\alpha u, D_t^\beta u, D_t^\alpha D_t^\alpha u, D_t^\alpha D_x^\alpha u, D_x^\beta D_x^\beta u, \dots) = 0, \quad 0 < \alpha, \beta < 1 \tag{3}$$

Where  $u = u(x, t)$  is an undefined function, following is the traveling wave variable...

Principal

$$u(x, t) = U(\zeta), \zeta = \frac{Tx^\beta}{\Gamma(1+\beta)} + \frac{ct^\alpha}{\Gamma(1+\alpha)} \tag{4}$$

Where  $\Gamma$  is an nonzero arbitrary constants, with using chain rule....

$$D_t^\alpha u = \sigma_u' \frac{dU}{d\zeta} D_t^\alpha \zeta, D_x^\alpha u = \sigma_x' \frac{dU}{d\zeta} D_x^\alpha \zeta \tag{5}$$

Where,

$\sigma_u'$  and  $\sigma_x'$  are denote as sigma tables following equation 7, we can take it deprived of defeat of overview,

$$Q(U, U', U'', U''', \dots) = 0 \tag{6}$$

Where the principal denotes this derivation of  $\zeta$  Suppose that the solution of equation 9 can be expressed by most  $\frac{G'}{G}$  as follows in equation 8,9.

$$u(\zeta) = \sum_{i=0}^m a_i \left(\frac{G'}{G}\right)^i, a_m \neq 0 \tag{7}$$

Where,

$a_i (i = 0, 1, \dots, m)$  are constants, while  $G(\zeta)$  satisfies following second order

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \tag{8}$$

With  $\lambda$  and  $\mu$  are stable. The positive integer  $m$  can be determined by the uniform equilibrium principle in equation 6 by substituting equation 7 into equation 6, and equation 8 forcing all the orders together. Subsequently, if every sub-angle of the polynomial is reduced to zero, we get a set of algebraic equations for  $a_i (i = 0, 1, \dots, m)$ , and  $\lambda, \mu, \Gamma$  solving these equations; you can find various exact solutions of equation (3).

Algorithm expansion method of  $(G'/G, 1/G)$

We use linear ordinary differential equation for the second order....

$$G''(\xi) + \lambda G(\xi) = \mu \tag{9}$$

We choose

$$\theta = G'/G, \theta = 1/G \tag{10}$$

For simplicity of after equation of 9 and 10 we gives.....

The details of ordinary differential equation 9 we conclude the following three cases

Case 1 if  $\lambda < 0$ , the simple solution of ordinary differential equation 9 will be read

$$G(\xi) = A_1 \sinh(\sqrt{-\lambda\xi}) + A_2 \cosh(\sqrt{-\lambda\xi}) + \frac{\mu}{\lambda} \tag{11}$$



$$\psi^2 = \frac{-\lambda}{\lambda^2 \sigma} (\phi^2 - 2\mu\phi + \lambda), \sigma = A_1^2 - A_2^2 \tag{12}$$

Case 2 if  $\lambda > 0$  the simple solution of ordinary differential equation 9 will be read

$$G(\xi) = A_1 \sin(\sqrt{\lambda\xi}) + A_2 \cos(\sqrt{\lambda\xi}) + \frac{\mu}{\lambda} \tag{13}$$

And corresponding relation will be...

$$\psi^2 = \frac{\lambda}{\lambda^2 \sigma - \mu^2} (\phi^2 - 2\mu\phi + \lambda), \sigma = A_1^2 + A_2^2 \tag{14}$$

Case 3 if  $\lambda = 0$ , the simple solution of ordinary differential equation 9 will be read

$$G(\xi) = \frac{\mu}{2} \xi^2 + A_1 \xi + A_2 \tag{15}$$

And we have

$$\psi^2 = \frac{1}{A_1^2 - 2\mu A_2} (\phi^2 - 2\mu\phi) \tag{16}$$

The arbitrary constant  $A_1$  and  $A_2$ , Suppose the solution of ordinary differential equation is polynomial and can be expressed  $\phi$  and  $\psi$  in that form

$$u(\xi) = \sum_{i=0}^N a_i \phi^i + \sum_{i=1}^N b_i \phi^{i-1} \psi \tag{17}$$

Where  $G = G(\xi)$  is the second solution linear ordinary differential equation 9,  $a_i, b_i (i = 1, \dots, N)$ ,  $\lambda$  and  $\mu$  are constant and positive integers  $N$  can be balanced by the principle in ordinary differential equation 6. Employed in equation 18, using equation 6, using 11, and 13 the left of equation 6 can be expressed as a polynomial, and where the degree is not greater than  $N$ , the system of algebraic equations Solve algebraic equations and substitute the values of  $i$ ; we can obtain the travel wave solution expressed by the hyperbolic functions of Eq. (6). (18) Substitution in equation 6 with using equation 11 and equation 14 or equation 11 and 16 we obtain the travel wave solution of equation 9 expressed by trigonometric and rational functions 10 and 11.

**CONCLUSION:**

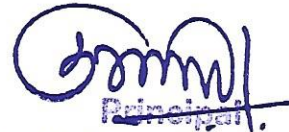
The methods  $(G' / G)$  and Extension Methods for Solving Nonlinear Fractional Partial Directional Equations These methods have their own advantages for nonlinear FDEs with fractional complex transforms: direct, concise, basic; And so it can also be applied to other FDEs where the uniformly balanced principle is Saint Ed.

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Jawahar Arts, Science & Commerce College  
Anadur, Tal. Tuljapur, Dist. Osmanabad.